

## **Supplementary material 1. Additional details on data analysis plan for assessing the model of synergistically interacting epidemics**

We employed multivariable linear probability regression models to assess the potential interaction on the additive scale. The linear probability model (with robust standard errors to correct for heteroskedasticity) was chosen over the logistic regression model because the former model is an additive model and the regression coefficients derived from the product terms can be directly interpreted as measures of interaction on the additive scale (*i.e.*, relative excess risk due to interaction or RERI) without further computational manipulation. Given that both models (linear probability and logistic) are valid statistically, and because the linear probability model is less computationally intensive, we opted to use the linear probability model instead of the logistic model. All comparisons of excess risk were in relation to those who reported no incarceration experience, severe violence victimisation, or needle/syringe sharing.

To assess interactions on the multiplicative scale, we used multivariable logistic regression models. After fitting these models, which estimated multiplicative interaction on the odds ratio scale, we applied Stata's margins command to estimate multiplicative interaction on the probability scale. Non-zero and statistically significant semi-elasticities indicate the presence of interaction on the multiplicative scale. These semi-elasticities can be interpreted as the proportional change in the expected outcome value associated with a one-unit change in the covariate. For instance, a semi-elasticity of 0.30 is interpreted as a 30 per cent relative change in the expected outcome for each unit increase in the covariate. For product terms of binary exposures, the semi-elasticity represents the percent relative change in the expected outcome (HIV-positive status) attributable to the interaction.

To ascertain that our findings were not driven by potentially arbitrary selection of dichotomisation cutoffs for the exposure variables, we conducted sensitivity analyses in which the needle/syringe sharing and physical violence victimisation exposure variables were not dichotomised for the multivariable logistic regression models. The continuous needle/syringe sharing variable ranged in value from 0 to 2 (0=no borrowing or lending of needle/syringe during the previous episode of injection drug use, 1=either borrowed or lent needle/syringe, and 2=borrowed and lent needle/syringe). The physical violence variable ranged in value from 1 to 5 (1=having experienced no physical abuse in the past 12 months, 2=once, 3=two to five times, 4=six to ten times, and 5=greater than ten times).